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Numerical Analysis of Concrete at Elevated temperatures

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ABSTRACT
Generally, concrete structures works at room temperature and this temperature will not fluctuate. But building fire accidents causes the structure in a building bearing a high temperature attack to reach maximum temperatures or even higher within a short time. As concrete is a conducting material, it conducts this elevated temperature and due to conduction temperature develops within the structure. The paper includes the development of coding for finding temperature distributed within concrete structure when subjected to elevated temperatures. Coding for non-linear transient heat conduction analysis will be generated and also a case study will be conducted. The work will be conducted for 2D state of temperature distribution. MATLAB software is used for coding and analysis.

***Keywords* -** Coding, Elevated, implicit, Non-linear, Transient.

I. Introduction

There are many practical engineering problems for which we cannot obtain exact solutions. This inability to obtain an exact solution may be attributed to either the complex nature of governing differential equations or the difficulties that arise from dealing with the boundary and initial conditions. When the transient temperature field of a structure under a fire accident or at elevated temperature is analyzed, the heating processes are variable, the nonlinear thermal parameters of the material vary with the temperature, and the boundary conditions are complex. Therefore, an accurate solution to the partial differential equation of heat conduction, which is non-linear is very difficult to determine. The equation of heat conduction is almost impossible to solve by the analytical method. The approximate analytical solution can be obtained only if the temperature field of the structure is simplified into a stable problem and constant values are used for the thermal parameters of the material. In order to deal with the nonlinear equation of transient heat conduction, the most effective and successful choice is to use the numerical method, i.e., finite difference or finite element analysis, and a high-speed computer to calculate the solution. The paper adopts a finite element method of numerical analysis. For the numerical analysis the software chosen is MATLAB.

II. Literature review

From the literature review, concrete specimens subjected to fire load can be broadly classified into three types namely Stressed, Unstressed, Unstressed residual strength tests. It is reported in the literature that behavior of normal strength concrete, high strength concrete etc. were different when exposed to high temperature. Many parameters influence the test results and affect the performance of concrete specimens exposed to high temperature. Since the test methods were costly and difficult to carry out, it is necessary to develop analytical modeling. Special attention has to be paid to the material properties for analysis and evaluation of the residual strength of structural elements exposed to accidental fire loading. Further examinations are needed in order to document material properties for design purposes and for the evaluation of residual strength of structural elements exposed to fire. Literature reveals that researchers adopt different procedures for the application of heat load as well as for testing the specimens. Hence there is a need to carry out an extensive investigation to find out the effect of the variations in the testing procedures.

III. Transient field problems

A variety of transient field problems, are governed by a differential equation of the following form;

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial u}{\partial z}\right)+pu+q=m\frac{\partial u}{\partial t} \qquad (1)$$

where kx, ky, *p, q, α* and m are known functions of (x, y, z). The solution variable u is a function both of space (x, y, z) and time t. The possible boundary conditions are as follows: (i) Essential boundary conditions: u specified. (ii) Natural boundary conditions- specified normal derivative along boundary;

$$k\frac{\partial u}{\partial n}\equiv\left(k_x\frac{\partial u}{\partial x}n_x+k_y\frac{\partial u}{\partial y}n_y+k_z\frac{\partial u}{\partial z}n_z\right)=\alpha u+\beta \qquad (2)$$

where α and β are specified parameters along the boundary. With respect to time, the differential equation is first order and therefore one initial condition is also needed in the following form;

$$u(x,y,z,t=0)=u_0(x,y,z) \qquad (3)$$

The general form of finite element equations for the problem can easily be written using exactly the same steps as those used for steady state problems. In order to use Galerkin's method, we need to develop an approximate weak form first. Assuming V refers to volume of an arbitrary element, moving all terms in the differential equation to the left-hand side, multiplying by the weighting functions Ni, and integrating over the volume, the Galerkin weighted residual for the problems is will obtain. Where dV=dx dy dz is the differential volume. Using the Green-Guass theorem on the first three terms in the weighted residual, we have;

$$\iint_S\left(k_x\frac{\partial u}{\partial x}n_x+k_y\frac{\partial u}{\partial y}n_y+k_z\frac{\partial u}{\partial z}n_z\right)N_i\,dS$$
$$+\iiint_V\left(-k_x\frac{\partial u}{\partial x}\frac{\partial N_i}{\partial x}-k_y\frac{\partial u}{\partial y}\frac{\partial N_i}{\partial y}-k_z\frac{\partial u}{\partial z}\frac{\partial N_i}{\partial z}+puN_i+qN_i-m\frac{\partial u}{\partial t}N_i\right)dV=0 \qquad (4)$$

where *S* is the surface of the element. On the surface there is a possibility of one of the two types of boundary conditions to be specified. Designating the surface over which u is specified as S_e and that over which its normal derivative is specified as S_n, the surface integrals can be written as follows;

$$\iint_S\left(k_x\frac{\partial u}{\partial x}n_x+k_y\frac{\partial u}{\partial y}n_y+k_z\frac{\partial u}{\partial z}n_z\right)N_i\,dS$$
$$=\iint_{S_e}\left(k_x\frac{\partial u}{\partial x}n_x+k_y\frac{\partial u}{\partial y}n_y+k_z\frac{\partial u}{\partial z}n_z\right)N_i\,dS+\iint_{S_n}(\alpha u+\beta)N_i\,dS \qquad (5)$$

Requiring the assumed solution to satisfy the essential boundary conditions results in weighting functions, that is zero over S_e. Thus with admissible assumed solutions the weak- form equivalent to the given boundary value problem is as follows;

$$\iiint_V\left(-k_x\frac{\partial u}{\partial x}\frac{\partial N_i}{\partial x}-k_y\frac{\partial u}{\partial y}\frac{\partial N_i}{\partial y}-k_z\frac{\partial u}{\partial z}\frac{\partial N_i}{\partial z}+puN_i+qN_i-m\frac{\partial u}{\partial t}N_i\right)dV$$
$$+\iint_{S_n}(\alpha u+\beta)N_i\,dS=0;\qquad i=1,2,\ldots \qquad (6)$$

Rearranging by keeping all terms involving unknown solution on the left-hand side, the weak form is;

$$\iiint_V\left(k_x\frac{\partial u}{\partial x}\frac{\partial N_i}{\partial x}+k_y\frac{\partial u}{\partial y}\frac{\partial N_i}{\partial y}+k_z\frac{\partial u}{\partial z}\frac{\partial N_i}{\partial z}-puN_i+m\frac{\partial u}{\partial t}N_i\right)dV-\iint_{S_n}\alpha uN_i\,dS$$
$$=\iiint_V qN_i\,dV+\iint_{S_n}\beta N_i\,dS;\qquad i=1,2,\ldots \qquad (7)$$

The assumed solution over an element is written as follows;

$$u(x,y,z,t)=\begin{pmatrix}N_1(x,y,z) & N_2(x,y,z) & \cdots & N_n(x,y,z)\end{pmatrix}\begin{pmatrix}u_1(t)\\u_2(t)\\\vdots\\u_n(t)\end{pmatrix}=N^Td \qquad (8)$$

where $u_1,u_2,\ldots$ are the unknown solutions at the element nodes that are functions of time. Note the interpolation functions N_i are functions only of space variables and are exactly the same as those used for steady state problems. Differentiating the assumed solution, we get;

$$\frac{\partial u}{\partial x}=\begin{pmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_n}{\partial x}\end{pmatrix}\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=B_x^Td$$

$$\frac{\partial u}{\partial y}=\begin{pmatrix}\frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_n}{\partial y}\end{pmatrix}\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=B_y^Td$$

$$\frac{\partial u}{\partial z}=\begin{pmatrix}\frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \cdots & \frac{\partial N_n}{\partial z}\end{pmatrix}\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=B_z^Td$$

$$\frac{\partial u}{\partial t}=\begin{pmatrix}N_1 & N_2 & \cdots & N_n\end{pmatrix}\begin{pmatrix}\dot u_1\\\dot u_2\\\vdots\\\dot u_n\end{pmatrix}=N^T\dot d \qquad (9)$$

where an over-dot indicates differentiation with respect to time. Substituting these into the weak form, we have;

$$\iiint_V(mNN^T)\,dV\,\dot d+\iiint_V(k_xB_xB_x^T+k_yB_yB_y^T+k_zB_zB_z^T)\,dV\,d$$
$$-\iiint_V(pNN^T)\,dV\,d-\iint_{S_n}\alpha(NN)^T\,dS\,d\quad=\iiint_V qN\,dV+\iint_{S_n}\beta N\,dS \qquad (10)$$

The three terms inside the second volume integral can be arranged in a more compact form by using matrices as follows;

$$k_xB_xB_x^T+k_yB_yB_y^T+k_zB_zB_z^T=\begin{pmatrix}B_x & B_y & B_z\end{pmatrix}\begin{pmatrix}k_x & 0 & 0\\0 & k_y & 0\\0 & 0 & k_z\end{pmatrix}\begin{pmatrix}B_x^T\\B_y^T\\B_z^T\end{pmatrix}\equiv BCB^T \qquad (11)$$

Where;

$$B^T = \begin{pmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_n}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \cdots & \frac{\partial N_n}{\partial z} \end{pmatrix} \text{ and } C = \begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{pmatrix}$$

Thus the finite element equations are as follows;

$$\iiint_V (mNN^T)\,dV\,\dot{d} + \left(\iiint_V BCB^T\,dV - \iiint_V (pNN^T)\,dV - \iint_{S_h} \alpha(NN)^T\,dS\right)d = \iiint_V qN\,dV + \iint_{S_n} \beta N\,dS \tag{12}$$

Define n x n matrices;

$$k_k = \iiint_V BCB^T\,dV; \qquad k_p = -\iiint_V (pNN^T)\,dV$$

$$k_\alpha = -\iint_{S_h} \alpha(NN)^T\,dS; \qquad m = \iiint_V (mNN^T)\,dV$$

Define n x 1 vectors;

$$r_q = \iiint_V qN\,dV; \qquad r_\beta = \iint_A \beta N\,dS$$

Thus the element equations are a system of first-order ordinary differential equations;

$$m\,\dot{d} + (k_k + k_p + k_\alpha)\,d = r_q + r_\beta \tag{13}$$

For time dependence problems, where temporal discretization is needed, it is effective to employ numerical solutions in the time domain to get the solution. Re-writing above equation in terms $\{T\}^t$ and $\{T\}^{t+\Delta t}$:

$$M\left(\frac{T^{t+\Delta t} - T^t}{\Delta t}\right) + K(\theta T^{t+\Delta t} + (1-\theta)T^t) = f \tag{14}$$

This is an implicit scheme with a backward approximation for the time term. This scheme is unconditionally stable and the accuracy of the scheme is governed by the size of the time step. This equation is applicable for the case where the thermal parameters are not changing with respect to time step.

3.1 Coding generated

Using the above descriptions MATLAB coding was generated. The coding for non-linear heat conduction analysis is as follows:

```
% Transient heat flow through a body using quad4 elements
h =input('enter heat transfer coeff');
tf =input('enter ambient air temp');
htf = h*tf;
kx =input('thermal conductivity x dirn');
ky =input('thermal conductivity y dirn');
rho=input('enter density of material');
Q =input('enter heat generated in the body');
ql =input('enter heat flux');
t0 =input('enter temp acting on surface');
cp =input('enter specific heat capacity');
nodes = [input('enter nodes')];
lmm = [input('enter element using node number')];
debc = [input('as row the nodes in which t0 acts')];
M=zeros(dof);K=zeros(dof);R = zeros(dof,1);
ebcVals=t0*ones(length(debc),1);
dof=length(nodes);
elems=size(lmm,1);
K=zeros(dof);
R = zeros(dof,1);
% Generate equations for each element and assemble them.
for i=1:elems
lm = lmm(i,:);
[m, k, r] = BVPQuad4Element(kx, ky, p, Q, nodes(lm,:));
M(lm, lm) = M(lm, lm) + m;
K(lm, lm) = K(lm, lm) + k;
R(lm) = R(lm) + r;
end
% Compute and assemble NBC contributions
for i=1:size(lmm,1)
lm = lmm(input('enter element'),:);
num =input('no. of sides having NBC in the elt')
for side=1:num
[k, r] = BVPQuad4NBCTerm(input('specify side'),input('alpha'),input('beta'), nodes(lm,:));
K(lm, lm) = K(lm, lm) + k;
R(lm) = R(lm) + r;
end
end
% Nodal solution
t = ones(dof,1)*tf;
D= NodalSoln(t, M, K, R, debc, ebcVals)
f=1;
Tg=D(:,f+1)-D(:,f)/tf;
time=input('enter the time step');
for f=1:(length(time)-1)
g=f;
% Variation in thermal parameters
d=pvarying( Tg, g, kx, ky);
S(:,f)=d(:,f+1);
soln=[D(:,1),S]
end
```

This MATLAB programming (coding) can be used for Non-transient state heat conduction analysis of two dimensional structures.

3.2 Case study

A case study was conducted using the coding developed. The problem shows the usual working condition of a building at elevated temperature. A concrete member of cross-section 200mm x 500mm, is heated. Only three surfaces of the member, i.e. both side surfaces and the bottom are heated; the top surface is not exposed. The thermal conductivity is 1.475 W/m °C, coefficient of heat transfer is 5.5W/m² °C. The temperature of the medium surrounding the heated surfaces of the member is taken as 300°C. The element type considered in coding is isoparametric quadrilateral element. The thermal parameters, nodal coordinate and its connectivity etc will be entered gradually after running the coding developed in MATLAB. Fig. 1 shows the analysis of case study in MATLAB.

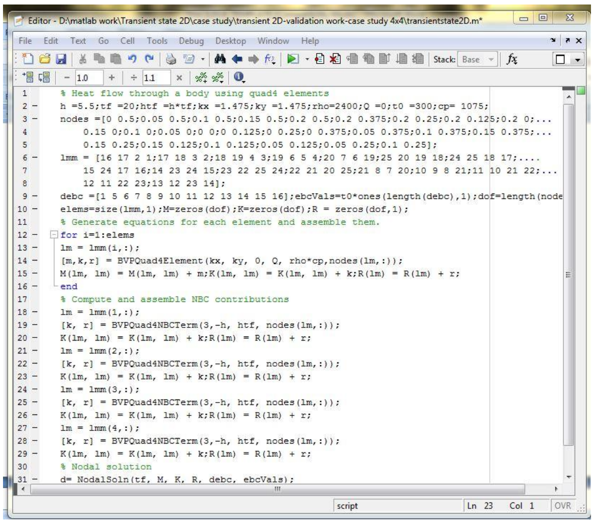

Fig. 1 Analysis of case study in MATLAB

The problem is also done using ANSYS 10.0 and the element used is Plane 55. In software we have only the provision to do a linear transient heat conduction analysis. In linear transient heat conduction analysis the change in thermal parametric values with time step is not considered. The results obtained thus will not show the usual working condition. This will be solved only by conducting a non-linear transient heat conduction analysis. Thus both non-linear heat conduction analysis and linear heat conduction analysis has been conducted by using different software and the temperature distribution has been obtained. The temperature distribution for a particular node is considered for comparison and the results found much variation. In order to analyze much serious practical problems, we have to go for non-linear heat conduction analysis.

IV. Conclusion

The MATLAB coding was developed for analysis of non-linear transient temperature field on the section of a concrete structural member based on the equations described above. The variable incremental time step, which is equivalent to equal increments of temperature of the surrounding medium, is used in the program, and it is helpful to ensure the stability of the calculation process. This program can be used widely for the analysis of a temperature field on the section of a structural member under various conditions of the temperature–time curve of fire. The computed results are compared with the solution of some special examples. For the analysis, high-speed computer is needed to calculate the solution for complicated structures. The coding is applicable for finding only the 2D state of temperatures distribution. Since the temperature distribution is not limited to a 2D state of distribution in a structure, extension of study is possible in this field.

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